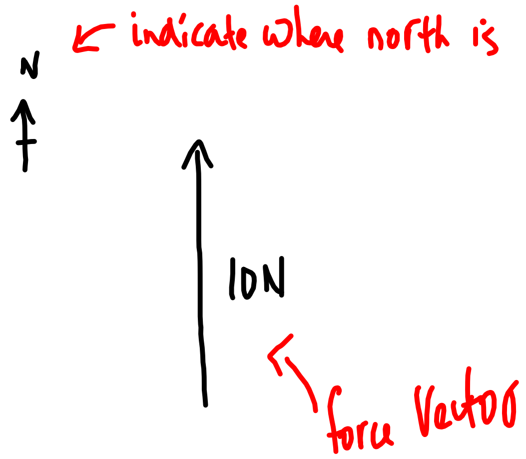
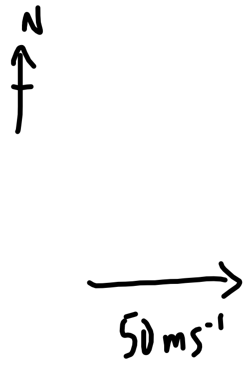
Representation of Vectors

$\vec{F}$ = 10N north
[N]



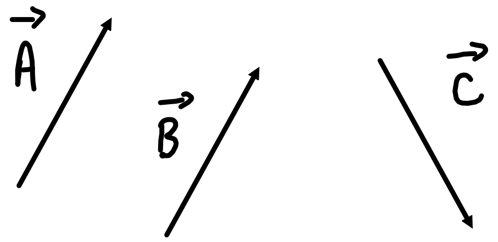
$\vec{v}$ = 50 m s^{-1} east
[E]



Equality of Vectors

$\vec{A} = \vec{B}$

(same size + same direction)



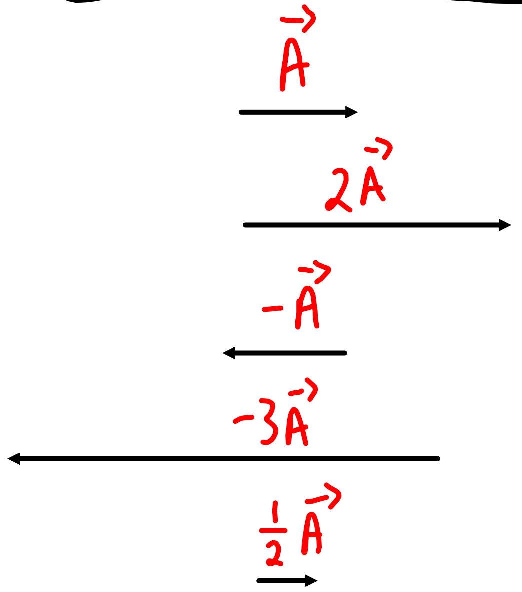
$\vec{A} \neq \vec{C}$

(same size but diff. direction)

$|\vec{A}| = |\vec{C}|$

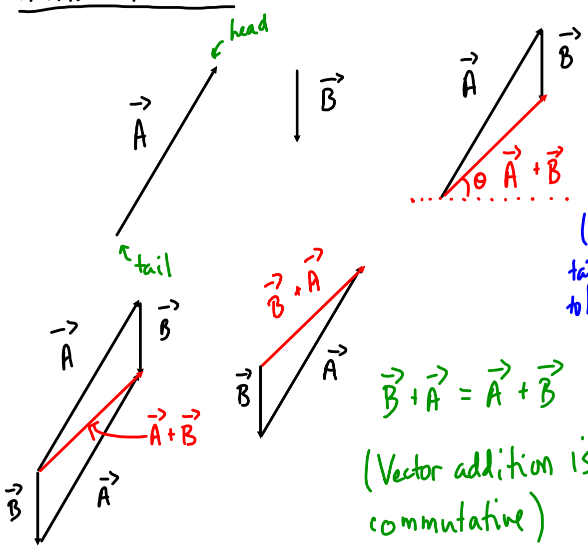
lines mean "magnitude"

Multiplication of a Vector by a Scalar:



- Direction stays the same unless multiplied by a negative scalar
- magnitude changes.

Addition of Vectors



join the vectors head to tail

(resultant tail of the first to head of the second vector)

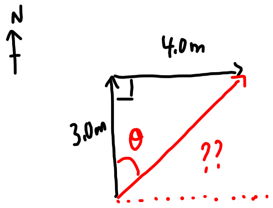
$\vec{B} + \vec{A} = \vec{A} + \vec{B}$

(Vector addition is commutative)

(i.e. the order does not matter)

Example

Find the sum of two displacement vectors 3.0m north and 4.0m east.



$c^2 = a^2 + b^2$

$c^2 = (3.0m)^2 + (4.0m)^2$

$c = 5.0m$ ← only the magnitude.

Direction: SOH CAH TOA

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{4.0m}{3.0m}$

$\theta = \tan^{-1}\left(\frac{4.0m}{3.0m}\right)$

$\theta \approx 53^\circ$ ← direction

The displacement is 5.0m [N53°E]

5.0m [E 37°N]

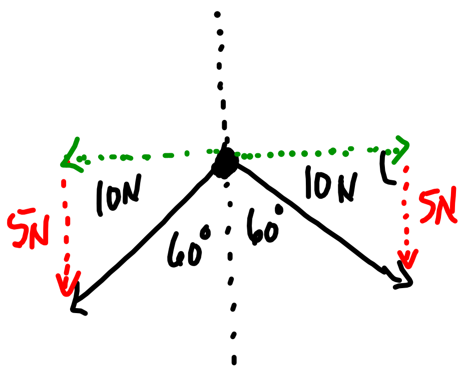
5.0m 53°T

↑ a true bearing of 53°

(For bearings, angles are measured CW)

Example

Two forces, each of magnitude 10N, act on a nail. One force is inclined downwards at 60° to the left of vertical and the other is inclined downwards at 60° to the right of vertical. What is the resultant force acting on the nail?



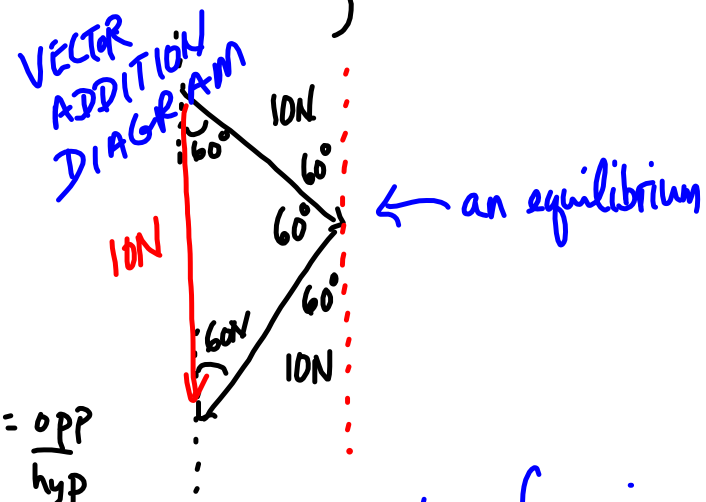
FBD (Free Body Diagrams)

(Note: you cannot add vectors here since not head to tail)

$\sin\theta = \frac{opp}{hyp}$

$\sin 30° = \frac{x}{10}$

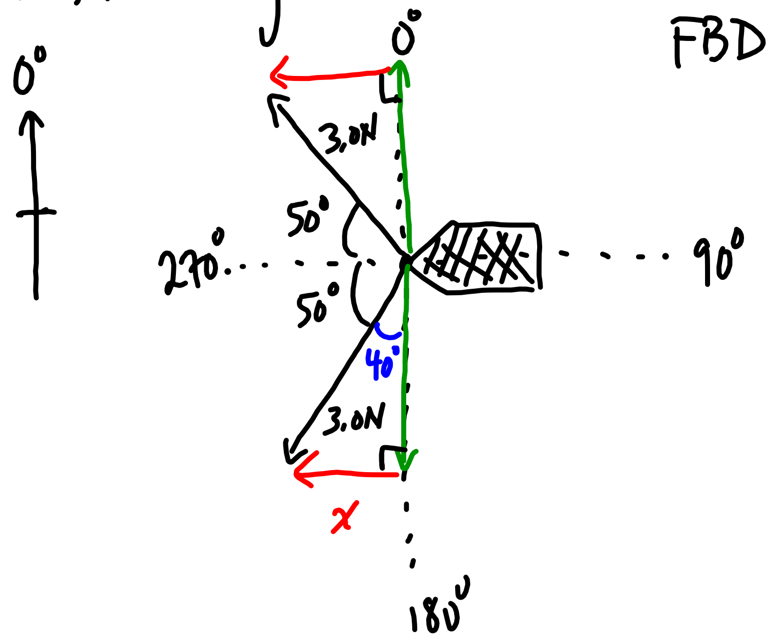
$x = 5\ N$



The resultant force is 10N [downwards]

Example

Two forces, each of magnitude 3.0 N, act on the front of a toy boat. One of the forces acts in a direction of 320°T and the other in a direction of 220°T. Determine the total force acting on the boat.



$\sin\theta = \frac{opp}{hyp}$

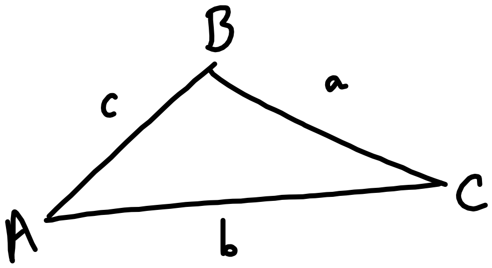
$\sin 40° = \frac{x}{3.0N}$

$x = (3.0N)\sin 40°$

$x = 1.9\ N$

So the resultant force is $2(1.9N) = 3.9N$
270°T

Non-Right Triangles



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

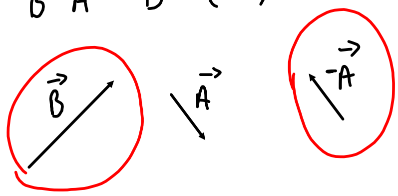
Law of Cosines

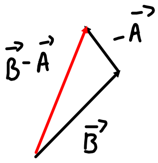
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Subtraction of Vectors

$5 - 3 = 5 + (-3)$

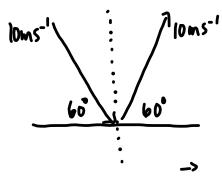
$\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$





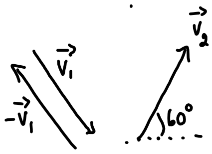
Example

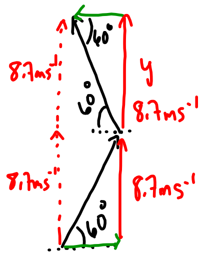
A billiard ball moving with a velocity of $10\,ms^{-1}$ inclined at $60°$ to the edge of the table bounces off the edge of the table at the same angle but with no change in speed. Determine the change in velocity of the ball.



$\Delta\vec{V} = \vec{V}_2 - \vec{V}_1$

$\Delta\vec{V} = \vec{V}_2 + (-\vec{V}_1)$





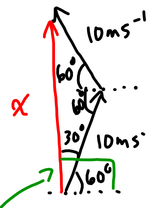
$\sin\theta = \frac{opp}{hyp}$

$\sin 60° = \frac{y}{10ms^{-1}}$

$y = (10ms^{-1})\sin 60°$

$y = 8.7ms^{-1}$

$\Delta\vec{V} = 17ms^{-1}$ directly away from edge.



because of symmetry this angle is $30°$ (straight away from edge)

Law of Cosines. or

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b}$

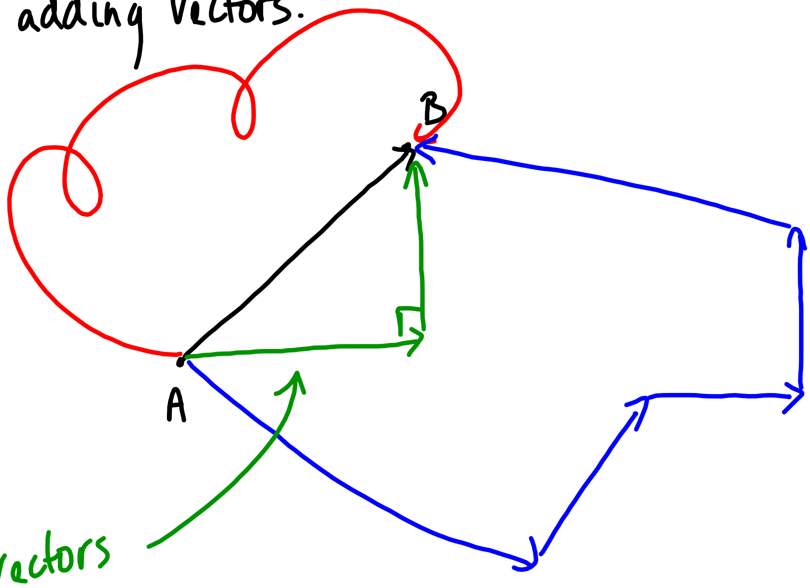
$\frac{\sin 30°}{10ms^{-1}} = \frac{\sin 120°}{x}$

$x = \frac{(\sin 120°)(10ms^{-1})}{\sin 30°}$

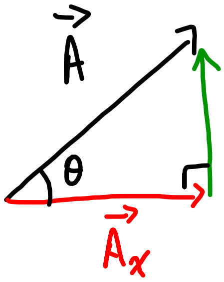
$x = 17ms^{-1}$

Components of Vectors

Resolving a vector into components is really just the opposite of adding vectors.



These vectors are components (they must be perpendicular to one another)



$\vec{A}_y$ (vertical component of $\vec{A}$)

$\vec{A}_x$ (horizontal component of $\vec{A}$)

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{A_y}{A}$$

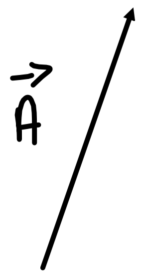
$$\cos\theta = \frac{adj}{hyp}$$

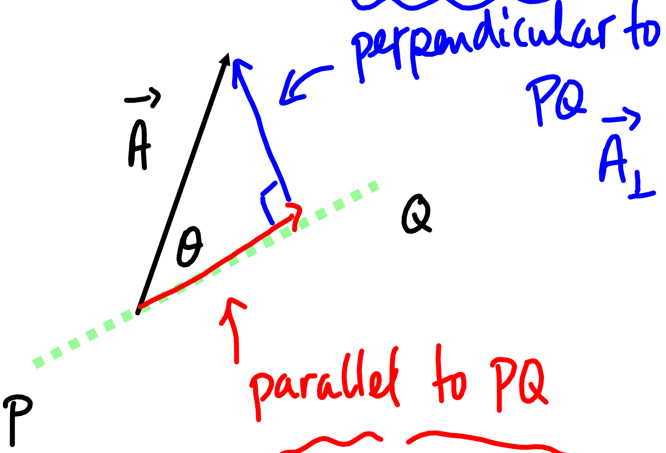
$$\cos\theta = \frac{A_x}{A}$$

$$\tan\theta = \frac{A_y}{A_x} \quad \frac{(opp)}{(adj)}$$

Resolving a vector into parallel and perpendicular components in relation to direction PQ.

$$A_{\perp} = A\sin\theta$$





$$A_{\parallel} = A\cos\theta$$

Example

A 10N weight is placed on a board which is inclined at an angle of 30° to the horizontal.

Determine the components of the weight acting down the incline (parallel to incline) and normal to the incline (perpendicular).